'Lived resources' and mathematics teachers professional development

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CREAD, University of Brest

Abstract

The resources available for mathematics teachers now proliferate. Traditional curriculum material is complemented by online resources of several kinds: software, lesson plans, even classroom videos. Each teacher can develop an individual website; online teachers associations propose digital textbooks etc. These evolutions impact the design and use of resources, the articulation between design and use; they position the teacher as a designer. Studying these issues with a research point of view requires a specific approach. I will present here a documentational approach developed for this purpose. I will illustrate the use of this approach by case studies concerning, on the one hand, individual teachers; on the other hand, teams of teachers involved in a teacher training program using a distant platform.

1. Digital resources for mathematics teachers

The starting point of our study is the increasing use, by mathematics teachers all other the world, of digital resources. Many kinds of such resources are available: software (which can be specifically designed for mathematics, like dynamic geometry systems, or not, like spreadsheets); digital textbooks; lessons plans; online exercises etc. Teachers communicate via e-mail, they discuss on forums. Schools develop Virtual Learning Environments (VLE), were teachers upload files for their students; the teacher-students out-of-class communication is more and more frequent.

While the digital means of communication and the online resources are widely used, the actual integration of software in class remains sparse, and far from the institutional expectations (Kynigos *et al.* 2007). Understanding the reasons for these gaps (between the wide use of online resources and the sparse use of software, on the one hand; between the institutional expectations and the actual use on the other) is a challenge, for the research about technology integration. It requires to take into account several factors, linked with the classroom environment, with the institutional context, and with the teachers professional knowledge and beliefs. Ruthven (2007) proposes to consider five structuring features of the classroom practice, influencing the integration of technology: working environment, resource system, activity format, curriculum script, and time economy. The “curriculum script” dimension in particular, describing “a loosely ordered model of relevant goals and actions which serves to guide their teaching” (Ruthven 2007 p. 61) belongs to professional knowledge. More generally, the recent evolutions of technology integration research position this issue within a wider professional development questioning, and indicate the need for comprehensive studies (Gueudet *et al.* 2010).

The evolutions of the available resources lead to develop new perspectives on their use by teachers. Teachers download files on websites; they combine several such files, associate them with texts they composed themselves. In some cases, these compositions will be uploaded by the teacher on a website, personal or shared with a group, an association. The teachers can not be positioned as passive users of resources: they are designers of their teaching. Moreover, design and use are strongly intertwined. Users can communicate with designers, and their suggestions can be quickly implemented.

Another important perspective evolution concerns the individual / collective articulation. The digital means offer new possibilities of communication. Teachers in different schools can discuss on forums. Even in the same school, the availability of a VLE facilitates the collaborative design of lessons.

Some of the evolutions evoked above correspond to new phenomena, new technical possibilities offered by the digital means. Nevertheless, most of them are perspective evolutions; the digital resources intervene as a lens, evidencing already existing facts. Even with traditional textbooks, teachers are designers. Even without VLE, teacher do not work alone. The focus on digital resources requires new research approaches, but these approaches do not only concern digital resources. They have to encompass all the interactions between teachers and various kinds of resources.

I present in part 2 such a theoretical approach, the documentational approach of didactics (Gueudet & Trouche 2009). This approach, and its consequences, are illustrated here by two case studies. The first case study (part 3) concerns a single teacher and her integration of technology. The second case study (part 4) considers a teacher training program, grounded in the collaborative design of lessons, using a distant platform.

2. Conceptualising resources and documents: a theoretical approach

The issue of digital resources for the teaching of mathematics leads to connect several theoretical perspectives, usually separated.

A conceptualisation of resources for mathematics teachers has been elaborated by Adler (2000). Her starting point was, in the context of South Africa, where teachers complain about the lack of elementary resources in many schools, to draw the attention on resources-in-use. She introduces a holistic definition of resource, suggesting to think of resource as a verb: anything likely to re-source the teacher's practice. This definition encompasses material resources, like the chalkboard; but also socio-cultural resources. Alder shows in particular that language, in the context of multilingual classes, can constitute an important resource for the mathematics teacher.

The focus on the interactions between teachers and resources is also central in the studies about curriculum material (Remillard 2005). These studies consider only material resources, designed for the teachers: textbooks, lesson plans, software... They identify adaptations by the teacher of the curriculum material. These adaptations can be outcomes of the teacher's preparation work; they can also happen in class, as a consequence of interactions with the students (Remillard distinguishes the “planned curriculum” from the “enacted curriculum”). In particular, they draw attention on the work of teachers in-class and out-of-class. The teachers modify the curriculum material they use; but the material also modifies the teachers practices, it shapes the teachers choices and can yield evolutions of the teachers professional knowledge. The research about curriculum material introduces the interactions between teachers and resources as an important component of teacher professional development.

Studying digital resources also leads to consider research about technology. The interactions with technological tools, and their consequences for knowledge evolutions, have been extensively studied in the case of students using various kinds of software, on computers or calculators. These processes have been in particular conceptualized within the instrumental approach (Guin *et al.* 2005), grounded in cognitive ergonomics (Rabardel 1995). Rabardel distinguishes an artefact, available for a given user, and an instrument, which is developed by the user, starting from this artefact, in the course of his situated action. These development processes, the instrumental geneses, are grounded, for a given subject, in the appropriation and the transformation of the artefact, for a given class of situations, through a variety of usage contexts. Through this variety of contexts, utilisation schemes of the artefact are constituted. A *scheme* (Vergnaud 1998) is an invariant organization of the activity, which comprises in particular rules of action, and is structured by *operational invariants* developed in the course of this targeted activity, in various contexts met for the same class of situations, and which pilot the activity. This definition can be represented by the equation: instrument = artefact+ scheme. This approach also distinguishes, within the instrumental geneses, two intertwined processes, the processes of *instrumentation* (constitution of the schemes of utilization of the artefacts) and the processes of *instrumentalisation* (by which the subject shapes the artefacts); this deep dialectical relationship between instrumentation and instrumentalisation

constitutes the core of the instrumental geneses.
Combining these different theoretical references, we introduced (Gueudet & Trouche 2009) a distinction between available resources, and a document, developed by the teacher in the course of her interactions with these resources, for a given objective.
This perspective – the documentational approach- is similar to the instrumental approach. The definition of a document can be represented by the equation: document = resources + scheme. The corresponding development process is called a documentational genesis. The geneses are long-term processes: schemes are developed across various contexts, encountered for a given class of situation. A class of situation, for a teacher, is a set of professional activities with a similar aim. For example, “preparing and setting up the introduction of functions”, “preparing and setting up applications exercises in algebra” are classes of situations, that the teacher encounters in different contexts (different classes, different years).
These geneses are naturally influenced by the institutional environment.

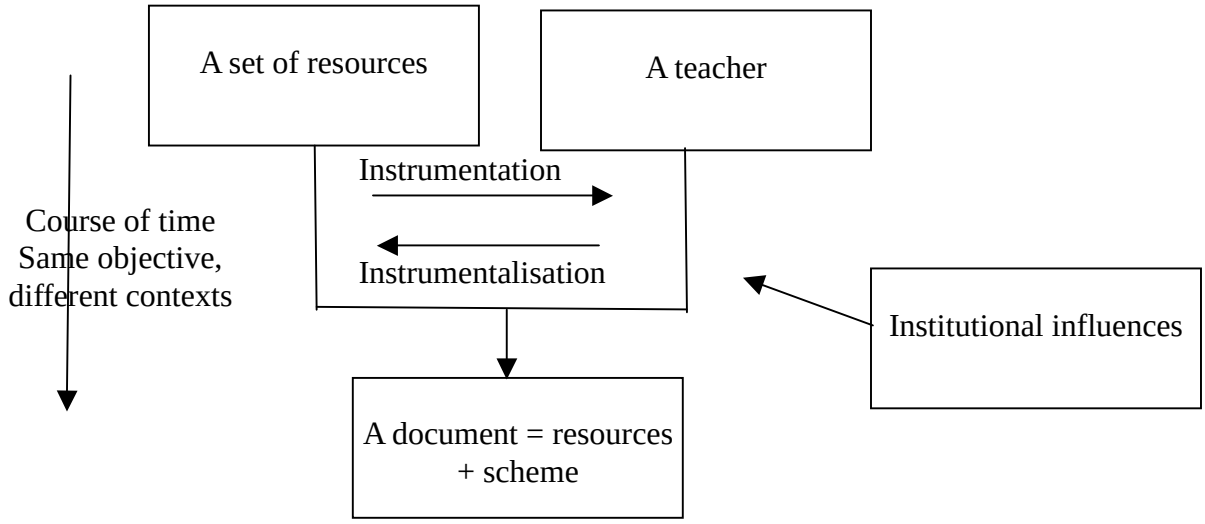

Figure 1. A documentational genesis

This figure represents a dynamic process. The document is not the outcome of a genesis: it is always evolving, during the teacher's preparation, during the implementation in class etc. This conceptualisation is not a simple application of the instrumental approach to the case of teachers. The resources, as defined by Adler, exceed indeed the artefacts. Naturally all artefacts are resources; but some resources, like a student's reaction, or the memory of a class discussion are not artefacts.
Resources intervene in all aspects of the teacher's work. What we call the documentation work: collecting resources, transforming them, setting them up in class etc. is central in the teachers professional activity. Thereof the documentational geneses hold a central place within teachers professional development.

I will now illustrate these considerations with a first example.

3. Teacher documentation: a case study

3.1 Studying teacher's documentation: methodology

The study of teachers documentation work requires a specific methodology. This documentation work happens indeed in many places, in-class and out-of-class; it involves different groups. The geneses are long-term processes, associating stabilities and evolutions; the teachers interact with multiple resources.

For this reason, we designed a methodology, entitled “reflexive investigation methodology” (Gueudet & Trouche to appear). The aim of this methodology is to follow, as completely as possible, a teacher's documentation work. This aim requires an active collaboration of the teacher followed: only the teacher herself has a complete access to her activity and resources.

For a given year, the follow-up lasts at least three weeks. During these three weeks:

- the teachers fills in a logbook, describing her professional activity (in-class and out-of-class), the resources used and produced, the agents involved;
- a lesson is observed and videotaped during the second week;
- three interviews of the teacher by the researcher are organised. Two interviews happen at the beginning of the follow-up: a general interview, about the teacher's resources, about her activity; a specific interview, about the preparation of the lesson that will be observed. The third interview takes place after the lesson observed, and is focused on the difference between what was planned and the enacted lesson;
- during the first interview, the teacher produces in particular a schematic representation of her resources system (SRRS), that can be complemented during the three weeks;
- during all the three weeks, the researcher collects as far as possible all the teachers' material resources: files, e-mails on the computer; written notes, students sheets etc.

In order to limit the amount of data, in particular the amount of description in the logbook, the follow-up concerns only one class.

To analyse these data, we associate quantitative and qualitative exploitation. We notice the different activities mentioned, and the time devoted to each activity; the resources intervening, the corresponding activities; the groups where the teacher is involved, the exchanges of resources with these groups.

In order to follow geneses, the data collection must not be limited to one academic year. We followed the same teachers in 2008-2009 and 2009-2010. During the second year, additional questions in the interviews concern the evolutions observed by the teacher, and the reason for these evolutions. The analysis of the data is also focused on these evolutions.

I will now present a case study, concerning a teacher, Myriam, followed during two years with this methodology.

3.2 Myriam's activity and resources

Myriam (51 years old) is an experienced teacher: she is teaching at lower secondary school (from grade 6 to 9 in France) for 29 years. We followed her in a grade 9 class (4 hours a week with the students in class). During the year 2009-2010, she filled in the logbook between the 1^{rst} of December and the 10th of January, which corresponds to 4 weeks of courses, and two weeks of Christmas holidays. During this period, she should have had her students in class for 4x4=16 hours; in fact 2 hours were cancelled because of the snow (we evoke below the consequences of this particular circumstance). So for this grade 9 class, she worked in class for 14 hours; her logbook mentions 12 hours of out-of-class work.

Myriam's school is in the countryside; it is relatively small. She has two colleagues in mathematics: one works half-time in this school, and half-time in an other school; the other works half-time. In both cases, the colleagues come to school for their courses, and go back home afterwards. There is no collaborative work in the team of maths teachers in Myriam's school, except for compulsory collaborative work, to prepare common assessment texts in grade 9. At the end of grade 9, the students have indeed an exam, the “brevet des collèges”; a blank exam is organised during the year for its preparation.

Myriam works nevertheless with other colleagues. She is involved in a “Reflection circle”, a group of 6 teachers of neighbouring schools. In this circle, the teachers discuss problem texts, share resources and experiences. Myriam has been a member of many such groups over the years, in particular in the IREM (Institute for Research on the Teaching of Mathematics), an institute for in-service teacher training, where the training takes the form of design of resources by groups of teachers, working with a researcher. Myriam is now herself intervening as trainer for in-service teachers.

On a more personal level, her family also intervenes in her professional choices. Her husband is teaching physics; they discuss a lot at home about the articulations between maths and physics, and Myriam pays attention to this dimension in her teaching, trying to emphasize as much as possible the applications of mathematics to other sciences. Her daughter entered upper secondary school in september 2008. Working with her, Myriam observes what is expected at this level; it leads her to change the content of her own courses, in particular in grade 9, to prepare the students for grade 10.

Myriam closely follows the institutional expectations. Every week, she downloads and reads the official ministry publication; she also discusses with the local maths inspectors. She is involved is her school in the assessment of the “B2i”: certification of the students skills on computers and Internet, which started to be compulsory in 2009.

Myriam uses many material resources, digital resources in particular. The table below presents her most important resources, digital and non-digital.

Digital	**Non-digital**
Classroom's computer, video-projector	Personal notebook
Students laptops	Class notebook
Official websites, official curriculum, mathematical situation proposed online on official websites	Planning notebook, agenda
Sesamath[1] digital textbook	Class textbook and other textbooks
Software: Open office (writer, calc); GeoGebra	Activity texts coming from in-service training followed as trainee
e-mail: professional address, mailing list of students	Professional newspapers
Calculators	Ring binder with the texts given during the previous years
Online exercises: MatouMatheux, Mathenpoche	Texts elaborated for the students
Software for managing the students marks: Profnote-Pronote	Students sheets
	Overhead projector

Table 1. Myriam's main material resources, digital and non-digital

The digital resources hold a important place in Myriam's resources, especially for her grade 9 class, because laptops are lent to the students for one year. During our two-years follow-up, we observed evolutions towards an even more important part of digital resources amongst her resources. In some cases, it is a consequence of institutional choices. For example, using the Pronote software is compulsory, for all the teachers in her school. All the students marks have to be filled in the software, which is used by the administrative staff to build the marks reports three times a year. In

1 http://www.sesamath.net. Sesamath is an association of mathematics teachers, developing free resources: a digital textbook, online exercises, a spreadsheet, a dynamic geometry system etc.

other cases, the changes correspond to Myriam's choices, following a discussion with a colleague, or an inspector, or a reading in a professional newspaper. For example, she ordered a webcam to replace her overhead projector, after reading, in such a newspaper, an article about the use of such a webcam in maths courses. I present in the next section two examples of evolutions observed, and their interpretations in terms of documentational geneses.

3.3 Evolutions in Myriam's digital resources: examples of geneses and documents

Introducing functions

During the two years of the data collection, we especially followed the teaching of a chapter devoted to the introduction of functions. At the same time, Myriam was starting to use Geogebra, and she used it in particular for this introduction of functions.

Figure 2 below presents an exercise text that Myriam proposed to her students. The starting point, for this text is an exercise proposed by a colleague of the "reflection circle". She has chosen to suppress all the intermediate questions.

Perimeter of a parallelogram inscribed in a right-angled triangle
ABC is a triangle, right-angled in A such that AB = 4 cm and AC = 3 cm.
M is a point of [AB].
The parallel line to (AC) through M intersects [BC] in N.
The parallel line to (BC) through M intersects [AC] in P.
Where should M be placed on [AB] to obtain a 9 cm perimeter for CNMP ?

Figure 2. Exercise text proposed by Myriam to her 9th grade students in 2008-2009.

Myriam's objective is to introduce the notion of function, the vocabulary "image", and the associated notations. The perimeter p of the parallelogram is such that $p = AM + 6$; Myriam wants to draw on this relation to introduce a function $p(x) = x+ 6$.
During the implementation in class, the text is written on the whiteboard; the students draw a figure on paper. Myriam uses the classroom computer and the videoprojector to display an animated Geogebra figure.

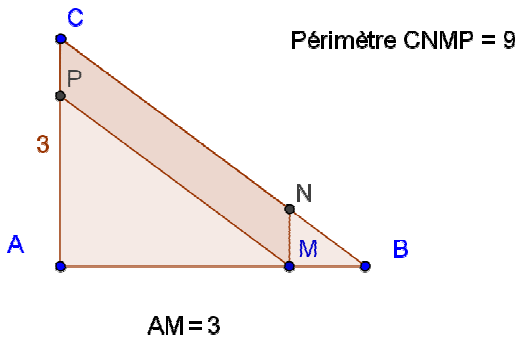

Figure 3. Geogebra figure, illustrating the perimeter

Myriam organises a classroom discussion, about the meaning of the question "where should M be placed?". The students quickly identify that if means determining the distance AM. Myriam displays this distance with GeoGebra, and the perimeter p of the parallelogram. She manipulates the animated figure, asking the students to observe when the value 9 is reached. The students observe that 9 is obtained when AM is "around 3 ". Myriam asks if they observed a link between p and AM, a student says "it is always + 6 !". She ask the students to prove this result on their sheets. Nevertheless, the proof is too difficult, and the students soon loose their focus, start discussing. Myriam is obliged to propose a detailed help for each step; the students hardly follow her

explanations during the end of the session.
In 2009-2010, Myriam chose not to use this activity, where the proof is too complex, and the usefulness of functions not clear. She still uses Geogebra during her first hour about functions, but this time to display a graph.
The function chapter was a new topic in the 2008-2009 curriculum. Myriam chose to use a situation requiring a modelling, and to use Geogebra to help the students to formulate conjectures. Myriam was an experienced user of Geoplan. In 2008, she started using Geogebra for several reasons: her colleagues, in the reflection circle, use it; her daughter use it at upper secondary school; and she finds it convenient to display functions graphs. She also uses it in geometry; but in all cases, she prefers to manipulate it herself, being not confident enough to leave the students use it.
With Geoplan, Myriam used to developed a document for the class of situations “preparing and setting up a proof activity for the students”. This document comprises a scheme, in particular an operational invariants like: “a phase of conjecture must precede the proof”, and “an animated figure is a good support for formulating a conjecture”. This knowledge guides her choice, for the new theme of functions and the new software, in an instrumentalisation process. However, in this case the proof following the conjecture was too difficult, and the students did not see its purpose. Thereof the following year, she retained a more simple introductory problem, around the theme of the volume of a rectangular box. The students build boxes themselves, it is more concrete, and no complex proof is required. In this case Myriam developed a document, about the introduction of function, comprising operational invariants like: “an introductory problem should be concrete, and should not require a complex modelling”.

Distant work with the students, development of a document

In 2008-2009, Myriam participated for the first time to the assessment of the computer certification (B2i, mentioned above). For this reason, she had to ask the students to send her e-mails with attached files. She created for this purpose a special e-mail address (we can consider this process as instrumentation). In 2009-2010, the heavy snow falls prevented the students to come to school for almost one week. Myriam used a students mailing list (especially elaborated by the administration to prepare school closing, in case of H1N1 flu!) to propose homework, in an instrumentalisation movement. She is only starting with such requests; so she does not give much precisions, about the name of the file to send back, or its format. She did not yet develop a stable orchestration (Trouche 2004, Drijvers *et al*. 2010) for such situations. The students sent back files with non-significant names; some send spreadsheets files, while others copy their graphs in a word processing file - in this case, Myriam can not see how they built their graphs. We consider that Myriam is developing a document, for the class of situations: “designing and setting up distant work about the graphs of functions”. The document has a “resource” part, associating in particular the classroom textbook; a spreadsheet; e-mail addresses, for the teacher and the students etc. Geneses are ongoing processes, associating evolutions and stability. Our observation took place at a moment of important evolutions, for this class of situations, linked with new digital means. We hypothesise that Myriam starts to develop a rule of action like “when asking the students to send spreadsheets productions, it is necessary to precise that the spreadsheet file itself must be sent, and not copied into a word processing”. This rule of action is associated with an operational invariant like: “correcting spreadsheet productions requires access to the formula written in the spreadsheet”. With the data we gathered, we cannot claim that Myriam actually developed this operational invariant; further observations are necessary to confirm this hypothesis. We consider it nevertheless as consistent, being connected with a more general operational invariant: “correcting the students mathematical exercises requires to have access to their procedures”, which seems to intervene in many documents developed by Myriam.

Myriam is one teacher, amongst several others followed with this methodology. Her case both illustrates the theory and confirms the relevance of a genesis perspective, and the importance of documentational geneses for professional development. Thereof in-service teacher training programs, especially directed towards the integration of technology have to take into account the

documentation work and associated geneses. We develop this issue in the next section.

4. Collaborative documentation work and professional development

With a documentation perspective, the evolutions of teachers practice and professional knowledge come from their interactions with resources, with a precise professional objective. Moreover, other research in the framework of the documentational approach have evidenced the importance of collaborative documentation, in communities of teachers (Gueudet *et al.* to appear). These statements lead to propose in-service teacher training programs grounded in collaborative design of lessons by teachers, the trainers proposing different kinds of resources. This choice is alos supported by the results of many other research works (see for example the different contributions in Krainer & Wood 2008). I present here such a training, and its consequences analysed in terms of geneses.

4.1 Collaborative design of lessons and blended teacher training

The program Pairform@nce (organised by the national ministry of education in France) proposes in-service training paths, aiming to sustain ICT integration for all school levels and all topics. These paths are templates for training programs to organize in the whole country. These training programs are blended: they are partly face-to-face and partly distant. They are grounded in the collective design of lessons, by teams of trainees (Gueudet *et al.* 2009).
All Pairform@nce paths comprise 7 stages. These 7 stages are not just successive steps; some of them are strongly intertwined. They are more like 7 different objectives of the trainers and trainees activity, during the training.
The first stage is the introduction, the beginning of the training. During this stage, the trainees meet each other, and meet the trainers. This introduction associates a face-to-face moment, and distant exchanges using e-mail, or a forum. For example, each trainee can load a personal presentation, and expose his or her expectations on such a forum. The second stage is the stage of elaboration of the teams, and selection of the topics retained; it is organised during a face-to-face workshop. Stage 3 is called “self and co-training”. During this stage, the trainees are trained, and train themselves, according to the path objectives. On the one hand they learn to use the ICT tools intervening in the path. But all paths also have, beyond the technical mastery, a more didactical objective. In the path I will consider here, in this stage the trainees learn about setting up investigation in mathematics, with dynamic geometry systems. The design of the lesson is step 4. The trainees, in the team, elaborate a content during their face-to-face meetings but also but through distant discussions, with the help of the trainer.
Stage 5 is the stage of classroom implementation. The lesson designed must be tested in class at least once; but several realisations are possible. Stage 6 is called “reflexive look back”. In this stage the trainees look back at the lesson designed, they reflect about what went wrong, what should be modified... It is quite frequent that a lesson does not work well for the first time; sometimes a classroom activity requires more time than what has been planned; sometimes the trainees must suppress some parts, because of time pressure. The final stage, number 7, is a stage of training evaluation. During this stage, the trainee fill in a questionnaire, explaining if the training met their expectations or not. They also formulate suggestions of modifications. The trainers also evaluate the training. They can send their remarks and suggestions to the path designers.

4.2 Investigation with dynamic geometry systems: example of a training path

In France, the official curriculum at lower secondary school invites to set up inquiry in class. It also invites to use ICT, in particular dynamic geometry systems (DGS). Nevertheless, in class DGS are not much used and investigation is not often organised. The training path “Investigation with dynamic geometry systems” faces thereof a double challenge. Its aim is that the trainees design and test a lesson giving real responsibilities to the pupils, regarding the use of the DGS and the mathematics at stake.

The training takes place during 13 weeks (outside holidays); it comprises three face-to-face workshops of one day each. Between these face-to-face days, a continuous work is done, using the e-mail, and the training distant platform.

The training starts with an e-mail contact, one week before the first face-to-face workshop. The trainers send attached to this e-mail a first questionnaire for the trainees. This questionnaire permits to collect the trainees expectations, to get informations about the material they can use in their school, and about their ideas and experiences about inquiry in class.

During the first workshop, the training is presented, the trainees teams are formed. The teams, if possible, comprise 4 teachers: 2 from one school, and 2 from another school. This way, the teachers will easily meet by pairs; but they will also be obliged to use distant communication, via the platform. This will ensure the regular connection to the platform, necessary to see the resources proposed by the trainers. Examples of lessons are presented. These examples permit to start a discussion that will go on during the whole training, by emphasizing important aspects of inquiry in mathematics:

- how is it possible to articulate inquiry, and the usual curriculum? How to avoid “loosing time”, regarding the mathematical content to be taught?
- which can be the link between investigation and proof, is there a risk that investigation hinders the learning of mathematics?

They also permit to present three grids, which will be used during all the training: grid for the description of a scenario; grid for observation of an inquiry-oriented session with dynamic geometry; grid for final report and suggestions about the lesson.

At the end of workshop 1, two mathematical situations are proposed by the trainers. Each team will have to choose one of these situations, and plan a scenario for implementing it in class, with an inquiry-based approach. This scenario is designed between the face-to-face workshops 1 and 2. This distant work uses a specific forum, and a folder for exchanging the files on the platform. The trainers support this distant work, but do not try to influence the decisions of the team.

During the second face-to-face workshop, the scenarios proposed by the teams are presented and collectively analysed, using the following categories, introduced by the trainers:

- a problem with a real stake, but possible to tackle;
- organisation of an appropriation phase, construction of an experiment,
- observation, formulation of conjectures;
- test of the conjectures, search for elements of proofs;
- debate, argumentation.

Moreover the role of the software is investigated: do the students build an experimentation with it? Do they only manipulate and observe? Is the computer used, for building the proof? All these elements yield discussions in the group of trainees, and modification propositions for the scenarios proposed.

After these discussions, the trainee teams start to design the lessons they will test in class. These lessons are elaborated and tested between workshops 2 and 3. A first version of the lesson is designed, exchanging on the preparation forum. This lesson is set up in class by one of the trainees; at least one other trainee observes the lesson and takes notes. Suggestions of improvements are formulated. If possible, another test, incorporating the modifications, is organized. New suggestions of improvements are formulated. The description of the lesson is uploaded on the platform, at least one week before the last workshop. During the third face-to-face workshop, the lessons are discussed, and propositions of further improvements are formulated.

4.3 Collaborative documentation and investigation with dynamic geometry systems

A training, corresponding to the « Investigation with dynamic geometry » has been implemented in 2008-2009. 32 trainees attended the training, which was organised by 3 trainers. In order to follow and analyse the trainees documentation work, all their distant exchanges on the platform, the files they sent, and their initial and final questionnaires where collected.

At the beginning of the training, the skills of the trainees concerning DGS were very diverse, from novice to expert. About investigation, none of them really experimented it in class. Even for expert teachers, the DGS was mostly used by the teacher to project animated figures.

During the second face-to-face workshop, the trainees have discussed a scenario proposed by the trainers, for a situation called “the river”. In this scenario, the trainers proposed to use a dynamic geometry figure as “black box”. This figure has been constructed by the teacher, but the construction has been concealed. The students have to manipulate the dynamic figure to discover how a given point was constructed, instead of finding themselves which point gives the solution.

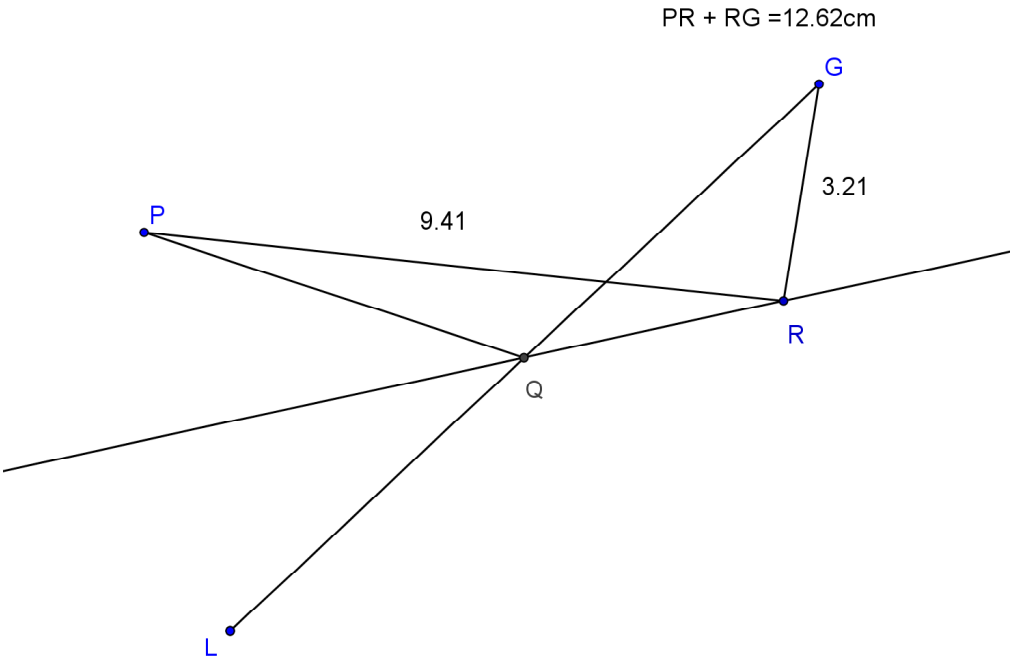

Figure 4. Find the point R on D such that PR+RG is minimal. The solution is given by the point Q.

This example has been chosen by the trainers to foster the discussion between the trainees: the use of dynamic geometry as “black box” can lead to interesting debates, as in the discussion below:

Trainee : In this example the students make trails and errors, in mathematics, they must learn logical reasoning!
Trainer: These trials and errors provide reasons for the logical reasoning which will be the next step!
Trainee: OK, but the computer is only used for manipulation. Maths are not physics! In physics you observe. It is just what they do here, they observe. It is not a deductive approach. I would not do that with my students, I want them to work with tools they know. When we tell “investigation”, what do we mean? When I want them to investigate in geometry, I want deductions.

This trainee seems to have developed an operational invariant like “solving a mathematical problem, for the students, means deducting”. During the same discussion, other trainees on the opposite have emphasized the interest of the black box, which is different from the usual practice. Students who usually encounter difficulties can feel more at ease with it; and it leads all the students to mobilize mathematical knowledge (here for example the properties of the reflection along a straight line).
Trainees have different opinions and experiences: thereof the common documentation work in the

teams fosters interesting discussions, likely to yield practices and professional knowledge evolutions.

9 teams of trainees were formed; the composition of the group of trainee did not permit to build the “ideal teams”, with 2 teachers in one school and 2 in another one. Only 4 teams comprised 2 teachers from the same school.
All the trainees have used, during the training, a DGS with their students in class. Most of them have tested the lesson designed in the team; some trainees, which did not have the appropriate class level, only set up initiation to the software. For all the trainees, the students themselves manipulated the software in a computer lab: this already constitutes an important evolution.

Amongst the lessons designed, 5 out of 9 concerned the introduction of new properties: area of the triangle, of the parallelogram, definition of cosine, angle at the center. The 4 other lessons corresponded to reinvestment of already known properties. In all these cases, the inquiry session was included in the normal progression. Avoiding to disturb the normal progression was very important for a majority of trainees. I do not claim that all the trainees developed an operational invariant like “investigation in class must not disturb the usual progression”; but this invariant prevailed in the choices, being shared by a majority of trainees.

The inquiry aspects remain limited. For 2 teams out of 9, the dynamic figure has been constructed by the teacher, and the students only have to manipulate it and observe the consequences of manipulation to formulate a conjecture. In 5 other teams, the students build themselves, or at least complement the figure. But the use of the figure is still limited to the formulation of conjectures. Only two teams have organised an investigation where the software is used also for the validation of the conjecture.

We can consider that, after the documentation work accomplished during the training, all the trainees share an operational invariant like: “constructing and manipulating a dynamic figure is useful to support the formulation of conjectures by the students”. This operational invariant was present for some trainees before the training; the common documentation work in the team has led the others to develop it. After the training, this operational invariant might become a component of one or several schemes. For example, for the class of situations “design and set up the introduction of a new theorem in geometry”, the trainees should enact a rule of action like “the students will use the DGS to formulate a conjecture”. A minority of trainees have developed an invariant like “A DGS can be used for the validation of an hypothesis”.

These observations remain limited to what happened during the training; they should be complemented by further observations, to investigate the sustainability of the evolutions of practice. We retain nevertheless that the collaborative design of lessons is a promising mode of teacher training.

5. Conclusion

I have presented here the elementary principles of the documentational approach. Many others important concepts have not been evoked, for the sake of brevity: documentation systems of a teacher, and their evolving structure; collective documentational geneses in communities of practice, for example. The approach is still evolving, and the attached methodology of reflexive investigation evolves simultaneously.
The starting point of the approach is the interest for digital resources. The examples presented here indicate clearly that our studies went far beyond this objective. Even with a focus on digital resources, studying teachers documentation requires to take into account sets of resources of different natures: textbooks, discussions with colleagues, students productions... At the same time, the evolutions brought by the digital resources are illuminated by the study of the teachers geneses. Teacher are designers of their own resources; groups of distant teachers can communicate for collaborative work; teachers and students communicate out-of-school.

All these evolutions evidence the need for new research approaches.
They also yield evolutions in the choices for teacher training programs, which have to take into account the position of the teacher as a designer. With such a stance, the teacher trainer has to support the trainees geneses, which can lead to sustainable changes of practice.

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